The Subtleties of Probability

The human brain, I've come to firmly believe, is improperly wired to cope with matters probabilistic.

Let me give you an example or two (or three).

Which of the following sequences is more likely to have been produced by 50 tosses of an unbiased coin?

- (b) НТТНИНТНИННТНТТНИНТТНТТНИНТТНИНТНИНННИНННИНТНТНТН

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Very few of you, I'm fairly certain, chose (c). And, you'd be right. I'm also confident that significantly fewer than one-third of you chose (b). Somehow (b), like (c), doesn't 'look' random enough, what with all those runs of heads and the intermittent tail pairs and triplets. Where's all the expected THTHTHT stuff?

You'll probably not now be surprised to learn that (b) was, in fact, produced by tossing an unbiased coin, (c) was produced by tossing a coin that was weighted to make it likely to land heads 60% of the time, and (a) was produced by someone trying to create a sequence that they thought might be produced by tossing an unbiased coin. This person's sequence contains far too many single head and single tail sequences and far too few runs of heads and runs of tails. In short, it's not random enough – an error we humans often make when we try to create what we think are plausible random sequences.

How about this one:

There are 3 bags. One contains 2 gold coins, another has 2 silver coins and the third contains one coin of each type.

You pick a bag at random, and without looking inside take out one coin. It's gold. What is the probability that the other coin in that bag is also gold?

I'll give you a moment to work out an answer ...

Most people are convinced the correct answer is one-half (indeed, many have written indignantly – at best – to websites and magazines, berating and belittling those who claim otherwise). Their reasoning goes something like this: obviously there are only two of the bags you could have selected, the bag with two gold coins in it or the bag with one gold and one silver coin. The only way for the non-selected coin from the selected bag to be gold is if you chose the gold-gold bag rather than the gold-silver bag. There's nothing to favour one over the other, so it's a 50:50 proposition. Thank you, move on, there's nothing more to see here. Right?

Actually, the correct answer is two-thirds.

Why? Well, selecting a gold coin is twice as likely if you've chosen the bag with the 2 gold coins as it is if you've selected the bag with one gold and one silver coin. So you must be twice as likely to have selected the bag with two gold coins as the bag with one. Probabilities must sum to one, so the probability of the gold-gold bag is two-thirds and that of the gold-silver bag is one-third. The two bags are not, as many of us feel compelled to assume, equally likely to have been selected given the information that we know (viz we plucked out a gold coin from the bag we selected).

This problem reminds of a gambling website on which the author has claimed that you should always bet on the team receiving points start because there are only three possible outcomes – the team giving start wins, the team giving start wins but not by enough to cover the spread, or the team receiving start wins outright – and two of those three outcomes are favourable to the team receiving start on handicap betting. So, he reasoned, teams receiving start are always 2/1 on favourites to win on handicap betting.

Bookies, though, will offer you evens, or near to it, on the team receiving start, thus apparently providing you with the amazingly undiscovered gamblatorial bargain of the century.

It'd be fun, don't you think, to find the guy who wrote this and offer him 2/1 odds on a coin landing on its edge? After all, it is one of the three possible outcomes ...

Finally, here's an example with a football flavour:

Two teams, A and B, are scheduled to meet one another twice during the upcoming season. You estimate that Team A's probability of winning Game #1 is 0.6, as is its probability of winning Game #2. Team B's probability for both games is 0.4 (ie we assume that there are no draws and that there's no home ground advantage – perhaps they're both played at Aurora).

What's the most likely outcome of the two encounters:

- (a) Team A wins both
- (b) Team A and Team B win one each
- (c) Team B wins both

Here, (a) seems like the obvious choice - but it's wrong. Surprisingly to most, the result described in (b) is over 30% more likely than the result described in (a). The probability that Team A wins both games is just over one-third (ie $0.6 \times 0.6 = 0.36$) and the probability that Team A and Team B split the results is almost one-half (ie $0.6 \times 0.4 + 0.4 \times 0.6 = 0.48$).

(If you want to prove this to yourself, here's one way to do it: Get two dice and roll them twice, noting the totals on both occasion. Repeat this process, say 100 times, or until you get convinced or bored, whichever comes first. Each such pair of rolls is equivalent to the two game sequence we discussed above. Assign the totals 7 to 12 to Team A, and 2 to 6 to Team B. This is equivalent to Team A having a win probability of 58.3% and Team B having a win probability of 41.7%, which is fairly close to what we assumed above. Count how often you get two totals of 7 or more in your two rolls versus how often you get one total of 2 to 6 and another of 7 to 12. You should get the first result about 34% of the time and the second about 49% of the time. Alternatively, just take my word for it).

It's only once a team has a win probability in excess of two-thirds that it's more likely that it'll win both games than it is that it will split them, despite it playing a clearly inferior opponent.

MAFL Funds: Latest News

Just a reminder that the Heritage Fund will operate from Week 1 of the season proper, so Investors will need to have at least the Heritage Fund portion of their money with me by 14th March. As I mentioned previously though I'd much prefer it if all Investors could have all money to me by that date.

Here is a cut-down version of the table from the last newsletter explaining the Funds that will operate this year and some of the details of their operation.

Characteristics of the MAFL Funds

	Heritage Fund	Alpha and Beta Funds ¹	Chi Fund	Line Fund
First round of wagering	Round 1	Round 4	Round 5	Round 5
Bet Types	Head-to-Head	Head-to-Head Head-to-Head		Line/Handicap
Team types wagered on	True home teams ² only (in games where this is one), otherwise either team Price on offer must be at least \$2.00	True home teams only	True home teams ² only (in games where this is one), otherwise notional home team only. Tipped margin of victory must be between 1.00 and 12.99 points	Teams receiving between 7½ and 20½ points start on line betting
Likely maximum bet (as % of original Fund size)	Games with a true home team: 10-12% Games with no true home team: 15-18%	15-20%	15-20%	15%
Recommended Weight	30%	20% (each Fund)	15%	15%

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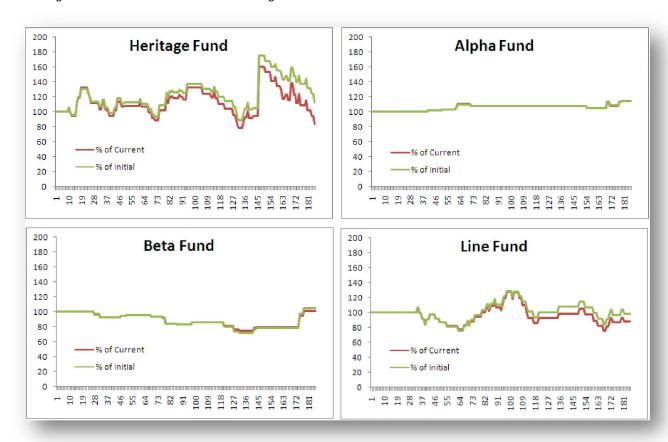
Please review the previous newsletter for more details on each of the Funds, including their real or simulated performances in seasons past.

This year, to simplify the post-round updates and to help Investors better track their returns, each Investor will be allocated a MAFL Investor Number (MIN) known only to him or her, and to me. I'll advise each Investor of his or her MIN prior to the start of the season. Returns to date and other performance data will then be reported in MAFL newsletters by MIN, protecting each Investor's anonymity and saving them from the need to work out which of the tabled results applies to their particular portfolio.

While we're on matters MAFL, one topic that continues to come up in correspondence with Investors is why our wagers are always a percentage of initial funds and not of current funds - in other words, why we don't bet more when we're ahead and less when we're behind. It's a good question and one worth us spending a little time on.

Determining the appropriate money management strategy is, for me, simply a question of returns. Whenever I've done the analysis previously, the returns from the 'Initial Funds' approach have always exceeded those from the 'Current Funds' approach considered over the same timeframe.

It's time then to conduct that analysis again. Let's take a look at the four Funds that were in operation last year. For each chart, the vertical axis measures the return on an initial 100 unit bank and the horizontal axis tracks game number, from 1 for the first game of the first round, to 185 for the Grand Final.



The charts show that, for each Fund, the Initial Funds approach proved to be superior at season's end to the Current Funds approach, such superiority varying in size from +34% for the Heritage Fund, through +12% for the Line Fund and +3% for the Beta Fund, to a tiny but nonetheless positive +0.03% for the Alpha Fund.

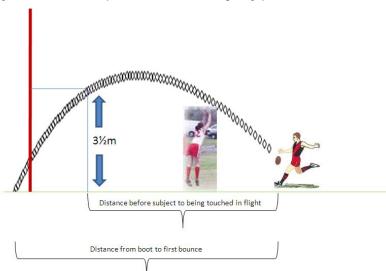
Furthermore, for the Heritage Fund, the Initial Funds approach led for 86% of the season and for the Line Fund, it led for 66% of the season. For the Alpha and Beta Funds the differences throughout the season are, frankly, too small to matter.

On the strength of this analysis, I think we'll stick with the Initial Funds approach this season. At some point during the season we will revisit it.

How Far Could You Kick a Footy on the Surface of the Moon?

Some of you, I'm sure, will have studied physics at some point in life and, in the course of that study, have completed a module called "Projectile Motion", or something similar. Few of you though, I'm guessing, have felt in any way compelled to revisit this topic since. Until now.

A size 5 Sherrin propelled from the boot of your favourite kicker is immediately subject to the laws of physics, which compel it to follow a vaguely parabolic arc towards its intended target.



Assuming you've had roughly the same level of exposure to physics that I had at school, you're wondering now why I said 'vaguely parabolic' in that last sentence, since all the projectiles we looked at back than followed precisely parabolic arcs. That's because, at school, we assumed away the existence of the air and the resultant drag-force on objects flying through it, such as a footy. The convenience of knowing that a kicked ball or any other propelled object will follow a simple-to-describe trajectory seems, to me, a steep price to pay for the absence of a breathable atmosphere.

And, as it turns out, the drag on the football caused by that breathable atmosphere matters a lot.

Based on biomechanical analyses of top-class AFL footballers, an average kick leaves the boot at just over 120 km/h and at an angle of about 50°. Such a kick, in windless (but not airless) conditions will travel about 56½m before bouncing, and about 55m before dipping below a height of 3½m at which point it becomes subject to being touched by a tall, leaping fullback with arms fully extended, Gadget-like, above his head.¹ When it arrives at the fullback it's still travelling at about 70km/h in the direction of its motion.

If we were to ignore the effects of drag, we'd predict that this same kick would travel about 116m, which would make the SCG an even more ludicrously small venue for AFL than it is already is.

As I said, accounting for drag is important.

Now let's take a closer look at that kick.

To clear the man on the mark the ball needs to be at least $3\frac{1}{2}$ m high as it passes over his head. To do this with a launch angle of 50° the kicker needs to contact the ball at least $2\frac{1}{2}$ m away from the marker.

If a kicker is looking to maximise the length of his kick, is 50° the best launch angle? Well, if we change the launch angle to 40° and repeat the analysis we find that the ball travels about 59½m from boot to bounce and about 57m before dipping below 3½m. So, with the shallower angle, the kicker would get about another 2m in distance to the point at which the ball first drops below 3½m. The

Comparison of a standard kick launched at 50° versus 40°

_	Launch Angle		
Selected Characteristic of Kick	40°	50°	
Boot to Bounce Dist	59½m	56½m	
Required launch distance from marker	4m	2½m	
Dist from Boot to first dip below 3½m	57m	55m	
Speed of Arrival when first dips below 31/2 m	65 km/h	70 km/h	
Hang Time	3.4s	4.0s	
Distance from marker to first dip below 3½m	53m	52½m	

problem is though, to clear the man on the mark, such a kick needs to be launched from about 4m away from him, and few kickers seem willing to add an apparent distance penalty of this size to their kicking routine. In truth though, they'd gain about ½m: 2m more in flight less 1½m for needing to be further back from the man on the mark.

¹ Assuming that the kick is taken at sea-level, with a standard ball weighing about 500g and sporting a 500mm circumference, kicked from 0.5m above the ground. Drag is assumed to be proportional to the square of the velocity in the direction of motion and the drag coefficient for a footy was obtained from *Flight of a Football (Aldis, Fulford, Weber and de Mestre; Mathematics and Computers in Sport, July 1992)*. Since you asked.

The shallower kick also arrives at a slightly slower velocity, around 65 km/h, and has a shorter 'hang time' – the time between being kicked and dipping below $3\frac{1}{2}$ m – of 3.4 seconds versus 4 seconds for the kick launched at 50° .

How about 30° then? Well that just won't work. Firstly, because the kick is so shallow, it needs to be launched from about 5½m from the man on the mark. Secondly, because it only travels 53m before dipping below 3½m. Relative to a 40° launch, that's a net loss of about 5½m. It does, though, have a shorter hang time: just 2.6 seconds.

More generally then, how do distance and hang time vary with launch angle and launch speed?

Distance ball travels relative to n	nan on the mark and ha	ang time before dipping below 3½m
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	Launch Angle									
_	30°		40°		45°		50°		60°	
Launch Speed	Dist	Hang	Dist	Hang	Dist	Hang	Dist	Hang	Dist	Hang
80 km/h	21½m	1.7s	28m	2.4s	29m	2.6s	29m	2.9s	30m	3.3s
90 km/h	28m	2.0s	34m	2.6s	35m	2.9s	35m	3.2s	31m	3.6s
100 km/h	34½m	2.2s	40½m	2.9s	41m	3.2s	40½m	3.5s	36m	3.9s
110 km/h	40½m	2.4s	46½m	3.1s	47m	3.5s	46m	3.7s	40½m	4.2s
120 km/h	46m	2.6s	52m	3.4s	52½m	3.7s	51m	4.0s	45m	4.5s
130 km/h	52m	2.8s	57½m	3.6s	57½m	3.9s	56m	4.25	49m	4.8s
140 km/h	57½m	3.0s	62½m	3.8s	62½m	4.1s	61m	4.5s	53m	5.0s

What strikes me about this table is how relatively small the variation is across a row (ie for a given launch speed) and how large it is down the columns (ie for a given launch angle). The difference between a 30° and a 45° launch angle seems, to me, fairly stark. Yet, for a given launch speed, it makes a difference of only 5 to 8m in distance.

Roughly the same difference is produced by varying the launch speed by just 10-15 km/h.

So, kickers struggling for distance would be better served working on their timing than on the angle at which the ball was leaving their foot.

Now, lastly, let's turn to the question of Moon-based kicking. That same kick that we first discussed – leaving the boot at 122km/h and with a launch angle of 50° – would behave markedly differently on the surface of la Luna compared to what it would do centre-ground at the G. On the Moon it'd travel almost 700m from boot to ground, taking almost 32 seconds to do so, arriving at its destination at a velocity of just 34 km/h despite having climbed to a height of over 200m. Which would make for one heckuva game of kick-to-kick.

That launch speed of 122 km/h seems very quick to me. To give it some context, I trawled the Internet a little (okay, a lot) to see what I could find on the speeds that objects move in other sports. The table on the right is the summary of what I uncovered.

It turns out that a kicked AFL football leaves the foot at about the same speed as a kicked soccer ball, which seems eminently logical.

More startling, however, are some of the other speeds, such as the 300 km/h badminton smash and jai-alai hurl, the 280 km/h golf drive and the 250 km/h squash forehand.

Surprising too is the similarity in the recorded maximum velocities for the pitched baseball and the bowled cricket delivery, particularly in light of the controversy that customarily surrounds any bowler who is considered to throw the cricket ball instead of bowling it.

Speeds in Sports

		Max Speed
Things that are	Sport and Action	(km/h)
Struck	Badminton - smash	302
	Golf - driver	282
	Squash - forehand	253
	Tennis - serve	224
	Baseball - batting	192
	Ice Hockey - slap-shot	168
	Polo - drive	161
	Indoor Volleyball - spike (men's)	130
	Table Tennis - smash	113
	Indoor Volleyball - spike (women's)	100
Kicked	Soccer	128
	AFL Kick	122
Thrown/Hurled	Jai-Alai	302
	Cricket (men's) - fast bowler	161
	Baseball - pitching	160
	Lacrosse	145
	Fast-pitch Softball - pitch (men's)	129
	Javelin	112
	Fast-pitch Softball - pitch (women's)	109
	Handball - throw	93
	Discuss	88
	American Football - throw	72
	Water Polo- penalty	59
	Shotput	53
	Tenpin Bowling	40

Pre-Season Bookmakers' Prices

It's been an interesting couple of weeks in the markets, not least because Sportsbet are now fielding a new market – on missing the Final 8 – that allows us to identify those teams for which Sportsbet feel they can charge a larger profit margin.

Here's how the various markets looked prior to this weekend's games, with the new market shown in the far right column.

		Bookmakers' Prices (29th Feb v					19th Feb)		
		Premiership Winner			Fina	al 8	Wooden Spoon		Miss 8
Team		TAB	Centrebet	Domebet	TAB	Centrebet	TAB	Centrebet	TAB
Geelong	П	3.00 L	3.15 S	3.25 -	1.06 -	1.05 -	301.00 -	251.00 -	7.00 -
St Kilda	W	7.00 S	9.00 S	8.00 S	1.30 S	1.33 S	101.00 L	81.00 -	2.60 -
Fremantle	L	11.00 L	11.00 -	10.00 -	1.33 L	1.42 L	101.00 -	81.00 -	2.85 -
Hawthorn	W	12.00 -	11.00 -	12.00 L	1.35 S	1.40 S	101.00 L	67.00 -	2.50 -
Port Adelaide	(L)	13.00 L	12.00 L	14.00 L	1.45 L	1.45 L	71.00 -	67.00 -	2.60 -
Collingwood	(L)	12.00 -	11.00 -	12.00 -	1.52 L	1.50 -	41.00 -	51.00 -	2.35 -
West Coast	(W)	12.00 -	12.00 S	14.00 -	1.48 L	1.55 L	31.00 -	26.00 -	2.50 -
Sydney	(W)	18.00 L	17.00 -	17.00 -	1.82 L	1.75 -	31.00 -	34.00 -	1.85 -
Adelaide	W	18.00 S	15.00 -	21.00 L	1.95 S	1.85 -	17.00 -	26.00 -	1.65 -
Brisbane Lions	(L)	23.00 -	21.00 -	23.00 L	2.25 L	2.00 -	18.00 -	21.00 -	1.50 -
Carlton	L	31.00 L	26.00 L	41.00 -	2.30 L	2.25 -	15.00 -	13.00 -	1.50 -
Kangaroos	(W)	31.00 -	34.00 L	41.00 L	2.50 -	2.75 -	8.00 -	8.00 -	1.40 -
Essendon	W	26.00 S	41.00 S	51.00 S	2.60 S	3.00 -	8.00 L	5.25 L	1.35 -
Western Bulldogs	L	41.00 L	34.00 -	41.00 L	2.60 L	2.75 -	6.00 S	8.00 -	1.42 -
Melbourne	(L)	51.00 -	51.00 L	51.00 -	3.60 L	3.50 -	3.80 S	5.00 S	1.18 -
Richmond	(W)	71.00 -	67.00 -	101.00 L	4.25 L	4.25 -	3.80 S	3.75 S	1.14 -
Over-round		21.0%	19.7%	11.4%	10.7%	9.5%	26.0%	21.6%	10.7%
W = NAB Cup Win, L = NAB Cup Loss, (W) = NAB Challenge WIN, (L) = NAB Challenge Loss									

(L = Lengthened, S = Shortened relative to 19th February prices)

What do I mean by 'profit margin' on a team? Well, consider the TAB's price for Geelong to make the 8 (\$1.06) and to miss the 8 (\$7.00). Converting these to probabilities gives 1/1.06 = 94.3% to make the 8, and 1/7 = 14.3% to miss the 8. These sum to 108.6%, so the bookie's overround is 8.6% and his theoretical hold is 1 - 1/1.086 or about 7.9%. This is the profit that he can expect to make with balanced action on both bets (ie amounts bet in proportion to the implied probabilities).

The table at right shows the equivalent percentages for all 16 teams in the competition. As you can see, they vary widely, from a low of about 7% for Port Adelaide to a high of about 13¼% for the Saints.

It's probably too early to read any significance into this considerable variability, especially given the fledgling nature of the Miss-the-8 market. It'll be an interesting phenomenon to keep an eye on though as the season unfolds.

(The weekend's matches provided no new information on this or on just about any other issue as the results produced virtually no change in any of the bookie's markets. The sole movement was St Kilda's shortening with Sportsbet in Premiership betting in to \$6.50).

	- f
	Profit
	Margin
St Kilda	13.3%
Hawthorn	12.3%
Essendon	11.1%
Melbourne	11.1%
Adelaide	10.6%
Kangaroos	10.3%
Richmond	10.1%
Brisbane Lions	10.0%
Fremantle	9.3%
Carlton	9.2%
Sydney	8.3%
Western Bulldogs	8.2%
Geelong	7.9%
Collingwood	7.7%
West Coast	7.0%
Port Adelaide	6.9%

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'til next time,

Tony

2 March 2008

Appendix Terms and Conditions for investing in any of the MAFL Funds

- 1. An investment into any Fund should be treated as 'spent' money. The return of any monies at the end of the season should be considered a pleasant surprise, and any return in excess of what you contributed should be heralded as a startling aberration in the space-time continuum.
- 2. Initially, shares can be bought in any Fund at \$1 per share. Over the course of the season, after each week's results I will recalculate the value of a share in each Fund. This will be the price at which you can then buy or sell shares in the Fund.
- The first wager for the Heritage Fund is likely to be on one or more of the matches in Round 1, which commences on March 20th. To participate in the fun for that round, you'll need to have money with me by Friday, 14th March, preferably earlier.
 - The first wagers for the Alpha and Beta Funds are likely to be on one or more of the matches in Round 4, which commences on April 11th. To participate in the fun for that round, you'll need to have money with me by the 4th April, preferably earlier.
 - The first wager for the Chi Fund is likely to be on one or more of the matches in Round 5, which commences on April 18th. To participate in the fun for that round, you'll need to have money with me by the 11th April, preferably earlier.
 - The first wager for the Line Fund is likely to be on one or more of the matches in Round 5, which commences on April 18th. To participate in the fun for that round, you'll need to have money with me by the 11th April, preferably earlier.
- 4. Each week you will be able to buy or sell shares, at the prevailing price, between Monday 9am and Monday 5pm. No transactions will take place outside that window.
- 5. For all Funds, the maximum bet on any one game will be 30% of the total initial funds provided by all participants. Generally, the bet on any one game will be substantially less than this figure.
- 6. Each Fund member will be notified of each Fund's bets for the round prior to the kick-off of the first game for the round. Each Fund member will also be notified of the Fund's financial status each week within 48 hours of the final game of the round, barring extraordinary circumstances.
- 7. If I've missed anything, I'll try to resolve any issues that arise in a fair and equitable manner. Ultimately though, the final decision and any associated karmic liability rests with me.

Season 2008